Using fuzzy non-linear regression to identify the degree of compensation among customer requirements in QFD

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ABSTRACT

As an effective customer-driven approach, the quality function deployment (QFD) takes numbers of customer requirements (CRs) into account in the process of the initial product design and the competitive analysis. It is a traditional multi-attribute decision making problem, and the trade-off strategy among CRs which is interpreted as decision parameters, is crucial for resulting the overall customer satisfaction. Although the general trade-off strategies concern about the importance weights of CRs, which are specified with a variety of methods, they ignore the influence of the degree of compensation among them. In this paper, we embed the degree of compensation among CRs into QFD, which is expressed as a symmetric triangular fuzzy number, and develop a fuzzy non-linear regression model using the minimum fuzziness criterion to identify it. Furthermore, an illustrative example is provided to demonstrate the application and the performance of the modeling approach. It can be verified from the experimental results that the overall customer satisfaction as well as the prioritization of products are affected by the degree of compensation among CRs. Meanwhile, against to the products in example, the overall customer satisfaction obtained with the traditional weighted-sum method is confirmed to be underestimated.

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1. Introduction

To improve the competitiveness and market shares, the worldwide companies increasingly concern about the voice of customers. The quality function deployment (QFD) was originated from Japan in the late 1960s [1]. As an effective customer-driven approach, QFD integrates customer requirements (CRs) into the product design to maximize the customer satisfaction with limited technical and resource constraints. The manipulation of QFD data can be expressed graphically in a matrix-like configuration called the House of Quality (HoQ) presented by Hauser and Clausing [11]. Until now, it has been successfully applied in many industries, such as software development process [5], supplier selection [3,4], electronics [15], R&D projects [34] and so on.

Generally, a number of CRs constituting the overall customer satisfaction for the product are taken into account in QFD, and the trade-off strategy among them that is finally reflected on the objective function is vital for the evaluation of the products and the acquisition of the optimal values of engineering characteristics (ECs). So far, the weighted-sum method has been one of the most commonly used trade-off strategies performing with the direct specification on different importance weights of CRs. Based on this idea, many methods have been developed, for example, AHP [2], FAHP [12,22], ANP [18] and FANP [16,27]. Besides, the fuzzy entropy method was also used to assess the importance weights of CRs [6,14]. Considering the differences in backgrounds, education, domain knowledge, etc., of the investigated customers, Wang [38] suggested that customers should express their preferences on the relative importance weights of CRs in their preferred or familiar formats.

However, as a typical multi-attribute decision making (MADM) problem, the trade-off strategies that are established in the above methods are incomplete. They only rely on the specification of importance weights of CRs and ignore other important decision parameters, e.g., the degree of compensation among them. Compensation refers to a willingness to allow high performance on one attribute to compensate for low performance on another and it is a property of a decision rather than a design [31]. In general, the degree of compensation is denoted with \( s \). It has long been certified that the weighted-sum aggregation of preferences cannot always identify all the Pareto points for a design and runs the risk of missing 'optimal' options with a default \( s=1 \). A family of aggregation operators \( P_s \) that governs the decision parameters involving both the importance weights of attributes and the degree of compensation among them

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was first proposed by Scott and Antonsson in 1998 [29] as follows, which spans an entire range of possible operators between min and max

$$
P_s(\mu_1, \mu_2, \ldots, \mu_n; \omega_1, \omega_2, \ldots, \omega_n) = \left( \frac{\alpha_1 \mu_1^\omega_1 + \alpha_2 \mu_2^\omega_2 + \cdots + \alpha_n \mu_n^\omega_n}{\alpha_1 + \alpha_2 + \cdots + \alpha_n} \right)^{1/s},
$$

(1)

where $\mu_1, \mu_2, \ldots, \mu_n$ are the attributes in the MADM problems, $\omega_1, \omega_2, \ldots, \omega_n$ are the importance weights of attributes with $\alpha_1, \alpha_2, \ldots, \alpha_n \geq 0$, and $s$ indicates the degree of compensation among attributes and ranges from $-\infty$ to $+\infty$. The aggregation operator $P_s$ possesses the property that every Pareto point could be the optimal solution through the different combination settings of decision parameters, $s$ and $\omega_i, i=1, 2, \ldots, n$, which collectively determine how to distribute the resources in the attributions to obtain the optimal objective with constraints. Meanwhile, four special situations are as follows:

$$
P_{-\infty} = \lim_{s \to -\infty} P_s = \min(\mu_1, \mu_2, \ldots, \mu_n),
$$

(2)

$$
P_0 = \lim_{s \to 0} P_s = \left( \frac{\alpha_1 \mu_1^\omega_1 + \alpha_2 \mu_2^\omega_2 + \cdots + \alpha_n \mu_n^\omega_n}{\alpha_1 + \alpha_2 + \cdots + \alpha_n} \right)^{1/0},
$$

(3)

$$
P_1 = \lim_{s \to 1} P_s = \frac{\alpha_1 \mu_1 + \alpha_2 \mu_2 + \cdots + \alpha_n \mu_n}{\alpha_1 + \alpha_2 + \cdots + \alpha_n},
$$

(4)

$$
P_{+\infty} = \lim_{s \to +\infty} P_s = \max(\mu_1, \mu_2, \ldots, \mu_n).
$$

(5)

Note that $P_0$ and $P_1$ are the forms of the geometric mean and the arithmetic mean, respectively, that we often used in the MADM problems.

Until now, the aggregation operator $P_s$ has attracted much attentions, especially from the area of engineering design. For example, in [31], the idea that both the degree of compensation and the distribution of importance weights among attributes must be considered to capture all potential acceptable decisions was illustrated by a simple truss design example. Kulok and Lewis [21] and See and Lewis [32] investigated the effect of different aggregation function formulations on multi-attribute group decision making. Scott [28] put forward an improved AHP method to quantify uncertainty in measurement error and different degree of compensation in trade-offs among criteria. The concept of the operator $P_s$ has been also used in the context of physical programming [25,26] that can successfully be integrated into both collaborative and multidisciplinary design optimizations [24].

Since the concept of the degree of compensation was proposed, the identification of $s$ value is always the main problem that needs to be solved at first. In 2000, Scott and Antonsson [30] applied indifference points to identify the value of $s$, but the selection of the indifference points is subjective, and finding two designs that are of exact equivalent value to a decision maker can be a challenging and time-consuming task [36]. Two attributes were included in the above application, so only three indifference points were needed for identifying the value of importance weights and the degree of compensation. Once the number of attributes increases, the process is hard to carry on. Besides, Chen and Ngai [9] presented a methodology combining the fuzzy set theory and the compensation strategy to optimize the target values of ECs and control the distribution of the development budget by varying the value of $s$. In [9], the degree of compensation $s$ was expressed as a crisp number and its determination which was on the basis of the engineering knowledge and experience of the decision makers was arbitrary and ad hoc. However, in our paper, considering that the degree of compensation among CRs is uncertain and imprecise, we would like to express it as a triangular fuzzy number, which is one of the most commonly used fuzzy numbers, in order to show how the overall customer satisfaction changes along with the change of the level of attainment for each customer requirement objectively. Meanwhile, utilizing the investigation of the overall customer satisfaction from customers, we develop a fuzzy non-linear regression model on the basis of the traditional fuzzy linear regression method, in which we set the minimization of the fuzziness as the objective function, constraining that all the observed values of overall customer satisfaction for the products must be involved in the $h$-level sets of the corresponding fuzzy outputs.

The rest of the paper is organized as follows. In the next section, the concept of the degree of compensation is embedded into QFD, and we build a fuzzy non-linear regression model to identify the degree of compensation $s$ among different CRs. In Section 3, an illustrative example is presented to demonstrate the proposed approach and a result analysis is given. Finally, some conclusions are drawn in Section 4.

### 2. Fuzzy non-linear regression model

Considering the deficiencies of the traditional weighted-sum method, which is usually set as the trade-off strategy in QFD, we introduce the concept of the degree of compensation among CRs into QFD and set the trade-off strategy as a combination together with the relative importance weights of them. In order to perform the uncertainty of the degree of compensation from customers, we express it as a fuzzy number. Furthermore, for ease of calculation, we set it as a symmetric triangular fuzzy number and then develop a fuzzy non-linear regression model to establish it.

#### 2.1. Problem description and notation

As an effective and widely applied method that transforms the CRs to the ECs, the main function of QFD is to assist the enterprises to proceed competition analysis and product preliminary design, the objective and principle of which are to improve the overall customer satisfaction of products as much as possible. Since the effectiveness of the objective function will directly impact on the products’ competitive analysis and market strategy, it is always an important issue in QFD. Based on the analysis of the deficiencies of the traditional weighted-sum method, we introduce the degree of compensation $s$ among CRs into QFD. Moreover, we express it as a symmetric triangular fuzzy number. And in order to establish the value of $s$ with the fuzzy non-linear regression method, the issues below need to be processed at first: (1) identification of CRs and their relative importance weights; (2) calculation and normalization of relationship matrix; (3) normalization of the values of ECs; (4) investigation of the overall customer satisfaction for given products; and (5) derivation of overall customer satisfaction, which will be described in the following subsections successively.

Before that, the notions that will be used are summarized as follows for reference:

- **CR** $i$ the $i$th customer requirement, $i=1, 2, \ldots, m$;
- **EC** $j$ the $j$th engineering characteristic, $j=1, 2, \ldots, n$;
- **Pro** the $p$th product, $p=1, 2, \ldots, k$;
- **$I_{pj}$** the value of EC $j$ of Pro $p$, $p=1, 2, \ldots, k$, $j=1, 2, \ldots, n$;
- **$x_{pj}$** the level of attainment of EC $j$ of Pro $p$ with $0 \leq x_{pj} \leq 1$, $p=1, 2, \ldots, k$, $j=1, 2, \ldots, n$;
- **$r_{ij}$** the strength of the relation measure between CRs and ECs, $i=1, 2, \ldots, m$, $j=1, 2, \ldots, n$;
- **$r_{ij}$** the normalized strength of the relation measure between CRs and ECs, $i=1, 2, \ldots, m$, $j=1, 2, \ldots, n$;
- **$R$** the relationship matrix between CRs and ECs with $R = \{r_{ij}\}_{m \times n}$. 

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\[ y_{pi} \text{ the level of attainment of CR}_i \text{ of Pro}_p \text{ with } 0 \leq y_{pi} \leq 1, \] 
\[ p = 1, 2, \ldots, k, \ i = 1, 2, \ldots, m; \] 
\[ y_p \text{ the vector of levels of attainment of CRs of Pro}_p \text{ with } y_p = (y_{p1}, y_{p2}, \ldots, y_{pm}), p = 1, 2, \ldots, k; \] 
\[ \tilde{y}_p \text{ the level of overall customer satisfaction obtained by the fuzzy non-linear regression; } \]
\[ w_i \text{ the relative importance weight of CR}_i, \ i = 1, 2, \ldots, m; \] 
\[ \bar{s} \text{ the degree of compensation among CRs; } \]
\[ h \text{ the fitting parameter in the fuzzy non-linear regression; } \] 
\[ g \text{ the number of investigated customers; } \]
\[ y_{opq} \text{ the level of customer satisfaction of Pro}_q \text{ investigated from customer } q, \ p = 1, 2, \ldots, k, \ q = 1, 2, \ldots, g; \] 
\[ Y_{op} \text{ the level of overall customer satisfaction of Pro}_q \text{ investigated from customers, } p = 1, 2, \ldots, k. \]

2.2. Identification of CRs and their relative importance weights

A variety of approaches have been proposed to identify CRs and their relative importance weights in the QFD literature, in which AHP is one of the most commonly used methods that can be employed to reconcile inconsistencies in managerial judgments and perceptions, and better resolve trade-offs. The AHP method treats the complex decisions as a system by reducing them to a series of pairwise comparisons and then synthesizing the results, so as to help decision-makers better organize their thoughts [37].

In 2002, Hsiao [13] used the combined AHP–QFD method to obtain the relative importance weights of CRs to aid the new product development. Similarly, in our paper, we apply the same method to identify CRs and their relative importance weights \( w_p \), \( i = 1, 2, \ldots, m \), in which three main operations are included. First of all, build the hierarchy for the overall customer satisfaction in graphical representation, which consists of all the CRs for the product. After that, construct a pairwise comparison matrix and calculate priority of each customer requirement. Finally, test the consistency of all judgments for the CRs and conduct the relative importance weights of them.

2.3. Calculation and normalization of relationship matrix

As an association of the CRs and the ECs of products, the relationship matrix is the essence of QFD. There are mainly two classes of methods to identify it: determining based on experts’ engineering knowledge and experience [9,23] and determining through the regression method, which is developed into the fuzzy linear regression method later [10,17]. However, when the fuzzy linear regression method is applied to estimate the relationship matrix in QFD which is constructed as a linear programming, the regression coefficients need to be turned into crisp numbers due to the characteristics of linear programming [33]. Afterwards, nonlinear programming-based fuzzy regression, considering both the center values and the spread values of the parameters estimated in the modeling phase, is developed [7]. However, no matter using the fuzzy linear regression method or the fuzzy non-linear regression method, the process is computationally expensive while the effectiveness is obscure.

In this paper, we focus upon the establishment of the degree of compensation among CRs, so we would like to employ the first method determining the relationship matrix between CRs and ECs based on experts’ engineering knowledge and experience. The strength of relationships is technically expressed with linguistic terms, such as weak, medium or strong, which are then translated into crisp numerical or predefined fuzzy numbers. In this paper, the strength of relationship \( r_{ij} \) is interpreted by the numerical scale 1–3–5–7–9 and normalized as follows:

\[ r_{ij} = \frac{1}{n} \sum_{j=1}^{n} r_{ij}, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \]  

where \( r_{ij} \) represents the normalized strength of relationship between CR and EC.

2.4. Normalization of the values of ECs

For going through the regression process, the values of all the ECs related to the CRs of products will be investigated in advance, while their properties, units and value ranges are extremely diverse. So in order to cover all types of inputs, the values of the ECs should be normalized to a scale [0,1], where 1 represents the total attainment of the given EC. Here, we classify the ECs into two types: the ‘smaller-the-better type’ (S-type) and the ‘larger-the-better type’ (L-type) [10], and then normalize them using the following transformation by taking Proq as an example:

\[ X_{pj} = \begin{cases} \frac{\ell_{pj} - \ell_{p}^{min}}{\ell_{p}^{max} - \ell_{p}^{min}}, & (S-type) \quad 7(a) \\ \frac{\ell_{p}^{max} - \ell_{pj}}{\ell_{p}^{max} - \ell_{p}^{min}}, & (L-type) \quad 7(b) \end{cases} \]  

where \( X_{pj} \) \( (j = 1, 2, \ldots, n) \) is the level of attainment of ECj of Proq with \( 0 \leq X_{pj} \leq 1 \), and \( \ell_{pj} \) \( (j = 1, 2, \ldots, n) \) is the value of ECj of Proq. Formulas 7(a) and 7(b) are for S-type and L-type ECs, respectively. For S-type ECs, \( \ell_{p}^{min} \) is the maximum value of EC that matches the performance of the main competitors and \( \ell_{p}^{max} \) is the minimum physical limit. Conversely, for L-type ECs, \( \ell_{p}^{max} \) is the minimum value of EC that matches the performance of the main competitors and \( \ell_{p}^{min} \) is the maximum physical limit.

2.5. Investigation of the overall customer satisfaction

One of the foundations of our fuzzy non-linear regression is the investigation of the overall customer satisfaction against to the given products from customers. The investigation results are indicated with \( Y_{op}, \ p = 1, 2, \ldots, k \), which will be used in the constraints of the regression model. The procedure to certain \( Y_{op} \) is described as follows.

Step 1: Let customer \( q \) evaluate Proq with one linguistic term, such as very perfect, general, very poor, and then translate it to a crisp number \( y_{pq} \) in \( 1, 2, \ldots, 9 \) with the principle that the better product gets the higher number, \( p = 1, 2, \ldots, k, \ q = 1, 2, \ldots, g \).

Step 2: Normalize the summation of \( y_{pq}, q = 1, 2, \ldots, g \), to acquire the overall customer satisfaction \( Y_{op} \) for Proq by

\[ Y_{op} = \frac{\sum_{q=1}^{g} y_{pq} - g}{9g - g} = \frac{\sum_{q=1}^{g} y_{pq} - g}{8g}, \ p = 1, 2, \ldots, k. \]  

2.6. Derivation of overall customer satisfaction

The introduction of the degree of compensation into QFD and the calculation by fuzzy non-linear regression are the main contributions of this paper. In the first place, considering the degree of compensation among CRs is uncertain and imprecise, we would like to express it as a fuzzy number. Additionally, the triangular fuzzy number and trapezoidal fuzzy number are two kinds of fuzzy numbers that are commonly used. So in this paper, we set \( s \) as a symmetric triangular fuzzy number for ease of
computing, and denote it as
\[ \tilde{s} = (s^l, s^u), \]
where \( s^l \) is the center value of \( 
\tilde{s} \), and \( s^u \) is the spread value. It is easy to deduce that the lower and upper limit values of \( \tilde{s} \) are
\[ s^l = \frac{s - s^u}{2}, \]
and
\[ s^u = \frac{s + s^l}{2}, \]
respectively. Furthermore, the membership function of \( \tilde{s} \) is known as
\[ u_\theta(x) = \begin{cases} \frac{|s^u - x|}{s^u - s^l} & \text{if } s^l \leq x \leq s^u \\ 0 & \text{otherwise.} \end{cases} \]

In QFD, assuming that the normalized strengths of the relation measure \( r_{ij} \) between \( C_i \) and \( E_j \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \), and the levels of attainment \( y_{pi} \) of \( E_j \) of \( P \), \( p = 1, 2, \ldots, k \), \( j = 1, 2, \ldots, m \), have been acquired according to Sections 2.3 and 2.4, the levels of attainment \( y_{pi} \) of \( C_i \) of \( P \) can be obtained with
\[ y_{pi} = \sum_{j=1}^{m} r_{ij} y_{pj}, \quad p = 1, 2, \ldots, k, \quad i = 1, 2, \ldots, m. \]

Subsequently, utilizing the trade-off strategy presented in this paper, which consists of the relative importance weights \( w_i \) of CRs and the degree of compensation \( s \) among them, the aggregation function of \( y_{pi}, i = 1, 2, \ldots, m \), representing the overall customer satisfaction of \( P \), denoted by \( \tilde{Y}_p \), can be expressed as
\[ \tilde{Y}_p = (w_1 y_{p1} + w_2 y_{p2} + \cdots + w_m y_{pm})^{1/\beta} = \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta}, \quad p = 1, 2, \ldots, k. \]

### 2.7. Development of the fuzzy non-linear regression model

Tanaka et al. [35] delineated a classical fuzzy linear regression model, the objective of which is to minimize the total fuzziness represented with the sum of spread values of fuzzy outputs under the constraints that all the observed values should be in the \( h \)-level set of the corresponding fuzzy outputs. Based on the underlying philosophy of the fuzzy linear regression in [35], we utilize the overall customer satisfaction \( Y_\rho \) investigated from customers and the fuzzy output \( \tilde{Y}_p \) to develop a fuzzy non-linear regression model as follows:

\[
\begin{align*}
\min \Delta &= \sum_{p=1}^{25} (Y_p^\rho - Y_p)^{1/\beta} \\
\text{subject to :} & \quad (\tilde{Y}_p)_l \leq Y_\rho \leq (\tilde{Y}_p)_u, \quad p = 1, 2, \ldots, k
\end{align*}
\]

(15)

where \( (Y_p)_l \) and \( (Y_p)_u \) are the lower and upper limit values of fuzzy number \( Y_p \) respectively, \( (\tilde{Y}_p)_l \) and \( (\tilde{Y}_p)_u \) are the lower and upper limit values of the \( h \)-level sets of \( \tilde{Y}_p \) respectively. Note that \( h(0 \leq h \leq 1) \) is the fitting parameter as a measure of goodness of fit, which is usually selected by the decision-maker.

In order to provide the specification of all the parameters in model (15), let us consider the fuzzy number \( \tilde{Y}_p \) first. It is obvious that the fuzzy output \( \tilde{Y}_p \) obtained by the exponent operations of the symmetric triangular fuzzy number \( \tilde{s} \) is no more a symmetric or triangular fuzzy number. So unlike the traditional fuzzy linear regression, the lower and upper limit values of \( \tilde{Y}_p \) cannot be calculated directly with the arithmetic algorithms of triangular fuzzy numbers. Generally speaking, it is not easy to obtain the arithmetic result of an exponential function of triangular fuzzy numbers. However, in our problem, it is fortunate that Scott and Antonsen [31] proved a proposition which shows that the aggregation function \( Y_p \), defined in (1) is nondecreasing as a function of \( s \). If we let \( F \) indicate the function of \( y_{pi} = (Y_{p1}, y_{p2}, \ldots, y_{pm}) \) and \( \tilde{s} \) in (14), i.e., \( \tilde{Y}_p = F(y_{pi}, \tilde{s}) \), it follows directly from the above proposition that \( F \) is nondecreasing with respect to \( \tilde{s} \).

Therefore, on the basis of the fuzzy extension principle proposed by Zadeh [39], we can obtain the lower limit value, the upper limit value, and the center value of \( \tilde{Y}_p = F(y_{pi}, \tilde{s}) \) as
\[
\begin{align*}
Y_p^\rho &= F(y_{pi}, s^l) = \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta}, \\
Y_p^\alpha &= F(y_{pi}, s^u) = \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta}, \\
Y_p^\circ &= F(y_{pi}, s^c) = \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta}.
\end{align*}
\]

(16) (17) (18)

respectively, where \( s^l \) and \( s^u \) are the lower and upper limit values of symmetric triangular fuzzy number \( \tilde{s} = (s^l, s^u) \) calculated by (10) and (11) respectively. Moreover, according to formulas (16) and (17), the objective function of model (15), i.e., the system fuzziness \( \Delta \) represented by the sum of spread values of the fuzzy outputs \( Y_p \), can be rewritten as
\[
\Delta = \sum_{p=1}^{25} (Y_p^\rho - Y_p^\circ) = \sum_{p=1}^{25} \left( \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta} - \left( \sum_{i=1}^{m} w_i y_{pi} \right)^{1/\beta} \right).
\]

(19)

Besides, the constraint (15b) in the regression model (15) indicates that the observed value of the overall customer satisfaction level of \( P \) from customers \( Y_\rho \) should be included in the \( h \)-level set \([ (\tilde{Y}_p)_l, (\tilde{Y}_p)_u ] \) of the fuzzy output \( \tilde{Y}_p = F(y_{pi}, \tilde{s}) \). Since \( F \) is nondecreasing with respect to \( \tilde{s} \), based on the interval arithmetic operational laws proposed by Kaufmann and Gupta [19], the lower and upper limit values of the \( h \)-level set of fuzzy output \( Y_p \) can be expressed as
\[
\begin{align*}
(\tilde{Y}_p)_l^\circ &= F(y_{pi}, s^l), \\
(\tilde{Y}_p)_u^\circ &= F(y_{pi}, s^u),
\end{align*}
\]

(20) (21)

respectively, where \( s^l \) and \( s^u \) are the lower and upper limit values of the \( h \)-level set of \( \tilde{s} \) respectively. It is known that \( \tilde{s} = (s^l, s^u) \) is a symmetric triangular fuzzy number, so the lower and upper limit values of \( h \)-level set of \( \tilde{s} \) are calculated as
\[
\begin{align*}
\tilde{s}_l^\circ &= s^l + hs^l, \\
\tilde{s}_u^\circ &= s^u - hs^l,
\end{align*}
\]

(22) (23)

respectively. According to formulas (20)–(23), the lower and upper limit values of the \( h \)-level set of fuzzy output \( Y_p = F(y_{pi}, \tilde{s}) \) are
\[
\begin{align*}
(\tilde{Y}_p)_l^\circ &= F(y_{pi}, s^l + hs^l), \\
(\tilde{Y}_p)_u^\circ &= F(y_{pi}, s^u - hs^l).
\end{align*}
\]

(24) (25)

Therefore, the constraint (15b) can be rewritten as
\[
F(y_{pi}, s^l + hs^l) \leq Y_\rho \leq F(y_{pi}, s^u - hs^l),
\]

i.e.,
\[
\left( \sum_{i=1}^{m} w_i (y_{pi})^{s^l + hs^l} \right)^{1/\beta} \leq Y_\rho \leq \left( \sum_{i=1}^{m} w_i (y_{pi})^{s^u - hs^l} \right)^{1/\beta}.
\]

(27)
In conclusion, based on (19) and (27), the fuzzy non-linear regression model (15) can be finally rewritten as

\[
\begin{align*}
\min \delta &= \sum_{p=1}^{k} \left( \left( \sum_{i=1}^{m} w_i^p \right)^{1/\delta} - \left( \sum_{i=1}^{m} w_i^\theta \right)^{1/\delta} \right) \\
\text{subject to} : &\quad \sum_{i=1}^{m} w_i^p \left( y_p(y_p)^{\theta} - h^\theta \right)^{1/\delta} \leq Y_op \leq \sum_{i=1}^{m} w_i^\theta \left( y_p(y_p)^{\theta} - h^\theta \right)^{1/\delta} . \\
&\quad p = 1, 2, \ldots, k.
\end{align*}
\]  

2.8. Flow chart of the proposed approach

Until now, the fuzzy non-linear regression model has been established, and we will show the process of problem-solving with the help of Matlab software in the next section. In summary, the proposed approach in this section can be depicted briefly by a flow chart (see Fig. 1). As shown in Fig. 1, in order to build the fuzzy non-linear regression model (15) can be finally rewritten as

\[
\begin{align*}
\min \Delta &= \sum_{p=1}^{k} \left( \left( \sum_{i=1}^{m} w_i^p \right)^{1/\delta} - \left( \sum_{i=1}^{m} w_i^\theta \right)^{1/\delta} \right) \\
\text{subject to} : &\quad \sum_{i=1}^{m} w_i^p \left( y_p(y_p)^{\theta} - h^\theta \right)^{1/\delta} \leq Y_op \leq \sum_{i=1}^{m} w_i^\theta \left( y_p(y_p)^{\theta} - h^\theta \right)^{1/\delta} . \\
&\quad p = 1, 2, \ldots, k.
\end{align*}
\]  

3. An illustrative example

In order to demonstrate the applicability of the proposed fuzzy non-linear regression model, an example of the development of a new type of motor car given by Chen et al. [8] is used and the results are presented and discussed in this section.

3.1. Construction of the HoQ

For identifying the CRs for the motor cars, a questionnaire survey is conducted to collect the voice from customers. After that, five major CRs are discovered and their relative importance weights are determined by the AHP method. Based on the design team’s experience and expert knowledge on this kind of product, five ECs that influence the five corresponding CRs are defined. At the same time, the relationship matrix \( R \) which denotes the relationship between ECs and CRs is calculated and normalized with the method given in Section 2.3. Besides, the values of ECs against to five products are also collected. Finally, the HoQ for the design of motor car is constructed and shown in Fig. 2.

As indicated in Fig. 2, EC1, EC3 are S-type and EC2, EC4, EC5 are L-type, so we can normalize them based on formulas (28(a)) and (28(b)), respectively, and the results are summarized in Table 1.

3.2. Analysis of the overall customer satisfaction

In order to obtain the customer satisfaction of the five motor cars, we investigate 1000 customers. Implement the steps given in Section 2.5 and the results are summarized in Table 2. We can see that Pro3 gets the highest customer satisfaction 0.805 while Pro4 gets the lowest 0.434.

<table>
<thead>
<tr>
<th>( w_i )</th>
<th>( EC_i^+ )</th>
<th>( EC_i^- )</th>
<th>( EC_i^\tau )</th>
<th>( EC_i^\delta )</th>
<th>( EC_i^\sigma )</th>
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<tr>
<td>0.31</td>
<td>CR1</td>
<td>0.8</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
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<td>0.25</td>
<td>CR1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>CR1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
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<td>CR1</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
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</table>

<table>
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<tr>
<th>Units</th>
<th>dB</th>
<th>Horsepower</th>
<th>Gallon</th>
<th>Kg</th>
<th>M³</th>
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<tr>
<td>Pro1</td>
<td>80</td>
<td>75</td>
<td>0.042</td>
<td>25</td>
<td>0.18</td>
</tr>
<tr>
<td>Pro2</td>
<td>65</td>
<td>65</td>
<td>0.034</td>
<td>24</td>
<td>0.20</td>
</tr>
<tr>
<td>Pro3</td>
<td>65</td>
<td>80</td>
<td>0.028</td>
<td>23</td>
<td>0.18</td>
</tr>
<tr>
<td>Pro4</td>
<td>75</td>
<td>60</td>
<td>0.032</td>
<td>16</td>
<td>0.14</td>
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<tr>
<td>Pro5</td>
<td>95</td>
<td>80</td>
<td>0.030</td>
<td>20</td>
<td>0.19</td>
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<td>0.027</td>
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<td>100</td>
<td>90</td>
<td>0.044</td>
<td>17</td>
<td>0.08</td>
</tr>
</tbody>
</table>

CR1: reducing the noise of the car;  
CR2: enhancing the acceleration;  
CR3: saving fuel;  
CR4: improving security;  
CR5: seat comfort;  
EC1: reducing the noise of the exhaust system;  
EC2: increasing the horsepower of the engine;  
EC3: reducing the amount of fuel per mile;  
EC4: increasing the controlling force of the braking system;  
EC5: enlarging the space of the seat.

Fig. 2. The HoQ for the motor car.
The overall customer satisfaction of motor cars investigated from customers.

<table>
<thead>
<tr>
<th>Products</th>
<th>Pro1</th>
<th>Pro2</th>
<th>Pro3</th>
<th>Pro4</th>
<th>Pro5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yop</td>
<td>0.542</td>
<td>0.700</td>
<td>0.805</td>
<td>0.434</td>
<td>0.578</td>
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</tbody>
</table>

Fig. 3. Change of the overall customer satisfaction of motor cars with the augment of s value.

Once the values of ECs, the relationship matrix, and the relative importance weights of CRs have been established according to the above procedures, the overall customer satisfaction of the product becomes a function of the degree of compensation s. We depict the graphs of these five functions (see Fig. 3) and list the overall customer satisfaction of the five products at several key points (see Table 3). Notably, the value of s here is crisp.

Furthermore, we can get different prioritizations of given products in the different intervals of the degree of compensation s as in Table 4, where the numbers denote the prioritization of products specifying '1' indicates that the product gets the highest priority, conversely '5' indicates that the product gets the lowest priority.

From Fig. 3 and Table 3, we can get the following conclusions:

1. The functions are monotonously increasing: the bigger the s value is, the higher the overall customer satisfaction is. When the value of s is small or large enough, the overall customer satisfaction of products is approaching the minimum or maximum values of ECs. For example, when s = -50, the overall customer satisfaction of Pro4 is 0.1452 and the smallest value of ECs is 0.1429, while when s = 50, the overall customer satisfaction of Pro3 is 0.9092 and the smallest value of ECs is 0.9412.

2. Pro3 and Pro2 always get a greater overall customer satisfaction than the other three products no matter what changes happen on the s value. The prioritization of Pro1 changes from the lowest to the third surpassing that of Pro4 and Pro2. Also we notice that the curves of Pro1, Pro4 and Pro5 intersect at the points marked with asterisk when s = -5.38 and 7.41. When the s value changes in the interval [-5.3800, 7.4050], the overall customer satisfaction of five products fluctuates markedly but their prioritization remains unchanged.

3. The overall customer satisfaction of five products resulted from the weighted-sum method which defaults the degree of compensation among CRs as s=1 is marked with circle.

4. Moreover, we give a comparison of the overall satisfaction of products shown in Tables 2 and 3. It can be seen that all the values of the overall customer satisfaction investigated from customers are lower than that calculated with s = 7.4050 and higher than that calculated with s = 1.

3.3. Fuzzy non-linear regression

After executing the above processes of customer investigation and analysis, all the data needed for the fuzzy non-linear regression model (28) have been collected. Based on the fourth conclusion in Section 3.2, we can narrow the range of s that will be established by the fuzzy non-linear regression down to [1.7,4.050] from (-∞, +∞). This will greatly reduce the computational effort of the following process, in which we operate the regression model to identify the degree of compensation s among CRs.

As for the fitting parameter h, the selection of a proper value of h is obviously important since it determines the ranges of the intervals. A higher h value yields wider spreads which imply a higher fuzziness [20]. As a measure of goodness of fit, it is usually selected by the decision-maker. In the earlier studies, the h value varies widely from 0 to 0.9. When the data set is sufficiently large, h could be set to 0, whereas a higher h value is suggested as the size of the data set becomes smaller [35]. In this paper, we execute the process of fuzzy regression with the data from five products and we set h=0.5.

The process to identify the degree of compensation s among CRs is designed as follows:

Step 1: Set Smin = 1, Smax = 7.4050, and h=0.5.

Step 2: Calculate the minimum feasible solution of the spread value s^ when the center value s^ changes from Smin to Smax. Then record the corresponding value of s^, s^ and the total fuzziness of five products.

Step 3: Identify the degree of compensation s that corresponds to the minimum total fuzziness of overall customer satisfaction.
Moreover, we give the curves (see Figs. 4 and 5) reflecting the change of the total fuzziness of five products and the spread value $s_S$ along with the increasing of the center value $s_C$. We can see that with the augment of $s_C$, the total fuzziness $\Delta$ and the spread value $s_S$ monotonously decrease at first and then monotonously increase, both of which reach their minimum when $s_C = 1.6250$. It is not difficult to understand that the total fuzziness $\Delta$, as a function of $s_S$ and $s_C$, has a similar trend with the spread value $s_S$. Based on the fuzzy non-linear regression model (28), the optimal solution is $\hat{s} = (1.6250, 0.3700)$ with the minimum total fuzziness $\Delta = 0.1386$, and $s_S = 1.2550, s^C = 1.9950$.

With regard to the five products, we figure out the trends of the membership degree and the fuzziness degree with the change of $s_C$ (Figs. 6–10). The trends of the fuzziness degree of products are consistent with the total fuzziness of the five products, while the trends of membership degree which monotonously increase at first and then monotonously decrease are just opposite to the total fuzziness of the five products. Each of the products arrives at their maximum membership degree $0.9986, 0.9973, 0.9978, 0.9984, 0.9982$ at $s_C = 1.4450, s_C = 1.8100, s_C = 1.6650, s_C = 1.4900, s_C = 1.5250$, respectively. Coincidentally, the five products reach their minimum fuzziness degree simultaneously at $s_C = 1.6250$. The points marked with asterisks and circles indicate the value of membership degree and fuzziness degree, respectively with the optimal solution (see Table 5). It can be seen from Table 5 that Pro$_3$ gets the lowest fuzziness degree 0.0048 and the highest membership degree 0.8892, Pro$_4$ gets the highest fuzziness degree 0.0536, and Pro$_2$ gets the lowest membership degree 0.5054.

Furthermore, we depict the membership functions of the fuzzy outputs of the five products with the optimal solution (see Fig. 11). Actually, since the spread value $s_S$ of $\hat{s}$ is small enough, the membership functions of the overall customer satisfaction of the five products are nearly triangular shapes. The points marked with asterisks stand for the membership degrees of $Y_{op}, p = 1, 2, \ldots, 5$ investigated from customers.

In summary, Table 6 shows the contrasts of the overall customer satisfaction for given products with different trade-off strategies interpreted by different combinations of decision parameters.
One trade-off strategy is generated with a combination of the relative importance weights and the degree of compensation among CRs acquired by our fuzzy non-linear regression model, whereas the other is generated only with the relative importance weights of CRs, which is via the conventional weighted-sum method. On one hand, we can see that the overall customer satisfaction calculated with our aggregation function (14) is a fuzzy number, so that it accords more with the characteristics of uncertainty and the imprecision of people’s judgements to products. On the other hand, all the center values, even the lower and upper limit values, are more close to the overall customer satisfaction investigated from customers than that obtained by the weighted-sum method, which means that the decision parameters of the overall customer satisfaction that we proposed in Section 2.6 are consistent with the relative importance weights of CRs, and the degree of compensation among them can more effectively represent the change of the overall customer satisfaction along with the change of the ECs. Actually, it can also be concluded that the traditional weighted-sum method underestimates the overall customer satisfaction of products. This kind of underestimation may lead to a misjudgment for companies to
make wrong assessments for their own and competitors’ products and make wrong decisions for market location or product design. Therefore, the value of $s$ is obtained, and the function of overall customer satisfaction for this kind of product can be definite regarding the relative importance weights of CRs. Afterwards, we can offer a more reasonable evaluation for the same kind of products or get the optimal ECs with constraints of technique and cost. In the end, during the practical applications of our approach, it should also be noted that (1) to different kinds of products, the degree of compensation among the customer requirements is different; (2) to one kind of products, the degree of compensation among the customer requirements changes over time.

4. Conclusion

In this paper, we embedded the degree of compensation among CRs into QFD, and due to its uncertainty, we expressed it as a symmetric triangular fuzzy number which is one of the most commonly used fuzzy numbers. In order to well identify it, we developed a fuzzy non-linear regression model based on the philosophy of the traditional fuzzy linear regression. The selection of optimal ECs and the competitive analysis for different products are two main applications of QFD, and they rely heavily on the trade-off strategy of CRs which refers to the decision parameters. In this paper, we used a parameter combination of the relative importance weights of CRs and the degree of compensation among them as the trade-off strategy to derive an objective function indicating the overall customer satisfaction. Furthermore, on the basis of the overall customer satisfaction investigated from customers, we established a fuzzy non-linear regression model to obtain the $\delta$ value. Finally, an illustrated example was presented to verify the application and the performance of the proposed fuzzy non-linear regression approach. The optimal solution $\delta = (1.6250, 0.3700)$ was acquired with the default constraint. Moreover, it was certified that the overall customer satisfaction resulted from our model could reflect the voice of customers more truly than that resulted from the traditional weighted-sum method. QFD practitioners can use the proposed methodology to identify the degree of compensation among CRs, and properly assess the strength and weakness of given products to formulate corresponding product improvement strategies.

After the identification of the degree of compensation $s$ through the fuzzy non-linear regression model has been done, how to determine the ECs to maximize the customer satisfaction in technical and resource constraints when considering the inner relationship between ECs will be a much valuable direction for future research. Besides, we can utilize the fuzzy non-linear regression model in the future to express the relative importance weights of CRs which were set as crisp numbers by the AHP method in this paper.

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References


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