Fuzzy random programming models for location-allocation problem with applications

Shuya Zhong, Yizeng Chen, Jian Zhou *

School of Management, Shanghai University, Shanghai 200444, China

ABSTRACT

Three types of fuzzy random programming models based on the mean chance for the capacitated location-allocation problem with fuzzy random demands are proposed according to different criteria, including the expected cost minimization model, the $\alpha$-cost minimization model, and the chance maximization model. In order to solve the proposed models, some hybrid intelligent algorithms are designed by integrating the network simplex algorithm, fuzzy random simulation, and genetic algorithm. Finally, some numerical examples about a container freight station problem are given to illustrate the effectiveness of the devised algorithms.

1. Introduction

The location-allocation problem plays an important role in project management since it was proposed by Cooper (1963) in 1963, which has been developed greatly in the past decades for its general practical application backgrounds, such as traffic network design (Hodgson, Rosing, Leontien, & Storrier, 1996), emergency logistics planning (Caunhye, Nie, & Pokharel, 2012), maritime logistics management (Choong, Cole, & Kutanoglu, 2002), and so on. Until now, researches on the location-allocation problem have been amply done for cases with deterministic demands, and many models and algorithms have been proposed for the problem in the literature (see, e.g., Kuenne & Soland, 1972; Ohlemüller, 1997; Wang, Li, & Zhou, 2014).

Considering the uncertainty of demands practically, studies towards the location-allocation problem have been expanded from the deterministic environments to the stochastic or fuzzy environments. On one hand, some stochastic programming models have been presented in Carriozza, Conde, Munoz-Marquez, and Puerto (1995), Sherali and Rizzo (1991), Zhou (2000), in which the demands of customers were assumed to be stochastic. In Zhou and Liu (2003) particularly, new models were given for the capacitated location-allocation problem with stochastic demands, and some hybrid intelligent algorithms, integrating the network simplex algorithm, stochastic simulation, and genetic algorithm, were produced to solve these models. On the other hand, various fuzzy programming models have been proposed when it is difficult to estimate the probability distributions for the demands of customers due to the absence of credible and accurate data. For example, Zhou and Liu (2007) formulated fuzzy programming models for the capacitated location-allocation problem with fuzzy demands including the expected value model, the chance-constrained programming model, and the dependent-chance programming model according to different decision requirements. Besides, many other fuzzy programming models as well as corresponding algorithms have also been presented for the location-allocation problem in Bhattacharya, Rao, and Tiwari (1993), Chen and Wei (1998), Liu and Zhu (2007).

Sometimes, the demands of customers may have the nature of randomness and fuzziness simultaneously. This stimulates many works being done on the location-allocation problem in such a mixed environment of randomness and fuzziness. In the view of random fuzzy programming, Wen and Iwamura (2008) proposed a random fuzzy $(\alpha, \beta)$-cost minimization model under the Hurwicz criterion. Subsequently, Wen and Kang (2011) discussed the location-allocation problem with random fuzzy demands, and presented three optimal models, i.e., the expected cost minimization model, $(\alpha, \beta)$-cost minimization model, and chance maximization model, to expand the research on the location-allocation problem under random fuzzy environments. In the field of fuzzy random programming, a fuzzy random facility location model with fuzzy random costs and demands was built by Wang and Watada (2012) to minimize the Value-at-Risk of the investment, by determining the optimal locations as well as the capacities of the new facilities to open with a hybrid modified particle swarm...
optimization approach. Moreover, in Liu and Xu (2011), the concept of hybrid variable whose instances were fuzzy random variable and random fuzzy variable was introduced, and a mixed-integer programming model was proposed for the location-allocation problem in a mixed environment of randomness and fuzziness, which was solved with a priority-based genetic algorithm.

In this paper, we make a further study on the capacitated location-allocation problem with fuzzy random demands based on the mean chance measure. Mean chance, initiated by Liu and Liu (2005b), is an index to measure the degree of the occurrence of fuzzy random events by using Choquet integrals. Taking advantage of the mean chance, this paper constructs three fuzzy random programming models including the expected cost minimization model, the x-cost minimization model, and the chance maximization model according to different criteria, and then solves them by some hybrid intelligent algorithms, which incorporate the network simplex algorithm and fuzzy random simulation into genetic algorithms.

The rest of the paper is organized as follows. In Section 2, some basic theories about fuzzy random variables are reviewed involving the mean chance and the expected value operator. In Section 3, a capacitated location-allocation problem is described, and parameters used are introduced. In Sections 4–6, three fuzzy random programming models are presented for the capacitated location-allocation problem in the fuzzy random environment. In order to solve these models effectively, some hybrid intelligent algorithms are designed in Section 7. Finally, some numerical examples about a container freight station problem is studied to illustrate the performance of the proposed algorithms in Section 8.

2. Fuzzy random theory

In the following, we briefly review the concepts of the fuzzy random variable, the mean chance of a fuzzy random event, and the expected value operator defined by the mean chance.

Fuzzy random variable is a mathematical description of a fuzzy random phenomenon. Kwakernaak (1978), Liu and Liu (2003b), and Puri and Ralescu (1986) have introduced the notions of fuzzy random variables according to different requirements of measurability. For our purpose, we use the following mathematical definition of the fuzzy random variable in our problem.

Definition 1 (Liu and Liu, 2003b). A fuzzy random variable is a function ξ from a probability space (Ω, A, Pr) to the set of fuzzy variables such that Pos{ξ(ω) ∈ B} is a measurable function of ω for any Borel set B of ℝ.

In order to measure the degree of the occurrence of a fuzzy random event, some definitions of the chance have been suggested, including the primitive chance (Liu, 2001a), the equilibrium chance (Liu & Liu, 2005a), and the mean chance (Liu & Liu, 2005b). In this paper, we adopt the concept of mean chance, which is a scalar index defined as follows.

Definition 2 (Liu and Liu, 2005b). Let ξ be a fuzzy random variable defined on the probability space (Ω, A, Pr), and fj : ℝ → ℝ continuous functions for j = 1, 2, . . . , q. Then the mean chance, denoted by Chj MX, of the fuzzy random event characterized by fj(ξ) ≤ 0, j = 1, 2, . . . , q, is defined as

\[
Ch^j_M \{ f_j(\xi) \leq 0 ; j = 1, 2, \ldots, q \} = \int_0^1 Pr \left\{ \omega \in \Omega | \text{Cr} \left\{ f_j(\xi(\omega)) \leq 0 ; j = 1, 2, \ldots, q \right\} \geq \beta \right\} d\beta
\]

where Cr is the credibility measure defined by Liu and Liu (2002).

Let

\[
F(\beta) = \text{Pr} \left\{ \omega \in \Omega | \text{Cr} \left\{ f_j(\xi(\omega)) \leq 0 ; j = 1, 2, \ldots, q \right\} \geq \beta \right\}.
\]

Then F(β) is a nonincreasing and left-continuous function with respect to β, and the geometric interpretation of the mean chance

\[
Ch^j_M \{ f_j(\xi) \leq 0 ; j = 1, 2, \ldots, q \} = \int_0^1 F(\beta) d\beta
\]

is shown in Fig. 1, which equals to the area of the shaded part.

Based on the mean chance in Definition 2, the expected value of a fuzzy random variable can be defined as follows.

Definition 3 (Liu & Liu, 2005b). Let ξ be a fuzzy random variable defined on the probability space (Ω, A, Pr). The expected value E[ξ] of ξ is defined as

\[
E[\xi] = \int_0^{+\infty} Ch^j_M \{ \xi \geq r \} dr - \int_{-\infty}^0 Ch^j_M \{ \xi \leq r \} dr.
\]

In addition, Liu and Liu (2005b) stated that the expected value E[ξ] in (4) can be rewritten as the following equivalent form given in Liu and Liu (2003b)

\[
E[\xi] = \int_0^{+\infty} \text{Pr} \left\{ \omega \in \Omega | E' \{ \xi(\omega) \} \geq r \right\} dr
- \int_{-\infty}^0 \text{Pr} \left\{ \omega \in \Omega | E' \{ \xi(\omega) \} \leq r \right\} dr
\]

where E' {ξ(ω)} stands for the expected value of the fuzzy variable ξ(ω), defined in Liu and Liu (2002) as

\[
E' \{ \xi(\omega) \} = \int_{-\infty}^{+\infty} \text{Cr} \{ \xi(\omega) \geq r \} dr - \int_{-\infty}^0 \text{Cr} \{ \xi(\omega) \leq r \} dr.
\]

3. Location-allocation problem

In this paper, the capacitated location-allocation problem considered is to find the optimal locations for a fixed number of facilities with capacity limitations in order to minimize the total transportation cost. We discuss the problem with assumptions that the path between any customer and facility is connected, and the unit transportation cost is proportionate of the supplied quantity and the travel distance. Each facility is restricted to be located within a certain region, and these potential locating areas have been observed in advance which are appropriate for constructing and operating the facilities. Besides, it is assumed that one facility
can serve several customers, and the demand of one customer can also be satisfied by more than one facility. Before modeling the capacitated location-allocation problem in the fuzzy random environment, let us introduce the following indices, parameters, and decision variables used in this paper:

- \( i = 1, 2, \ldots, n \): index of facilities;
- \( j = 1, 2, \ldots, m \): index of customers;
- \( (a_j, b_j) \): location of customer \( j \), \( j = 1, 2, \ldots, m \);
- \( \xi_j \): fuzzy random demand of customer \( j \), \( j = 1, 2, \ldots, m \);
- \( s_i \): capacity of facility \( i \), \( i = 1, 2, \ldots, n \);
- \( (x_i, y_i) \): decision variable representing the location of facility \( i \), \( i = 1, 2, \ldots, n \);
- \( z_{ij} \): quantity supplied to customer \( j \) by facility \( i \) after the fuzzy random demands are realized, \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, m \).

For convenience, we denote

\[
(x, y) = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \end{pmatrix}, \quad z = \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nm} \end{pmatrix}.
\]

Recall that in a deterministic location-allocation problem, both the location \((x, y)\) and the allocation \(z\) are decision variables, which are unchanged after they are decided. However, in the location-allocation problem with stochastic demands or fuzzy demands, the allocation \(z\) will be decided every period after the stochastic demands or fuzzy demands of customers are realized (see Zhou & Liu (2007)). Similarly, in our problem, if demands are supposed to be fuzzy random variables, the allocation \(z\) may be obtained after the fuzzy random demands are realized.

Suppose that the demands of customers are fuzzy random variables defined on the product space \((\Omega, \mathcal{A}, \mathbb{P})\times(\Theta, \mathbb{P}(\Theta), \mathbb{P})\). For each \((\omega, \theta) \in \Omega \times \Theta\), the value \(\xi_j(\omega, \theta)\) is a realization of fuzzy random vector \(\xi = (\xi_1, \xi_2, \ldots, \xi_m)\). An allocation \(z\) is said to be feasible if

\[
\begin{align*}
\sum_{j=1}^{n} z_{ij} &= \xi_j(\omega, \theta), \quad j = 1, 2, \ldots, m \\
\sum_{i=1}^{m} z_{ij} &\leq s_i, \quad i = 1, 2, \ldots, n \\
\sum_{i=1}^{m} z_{ij} &\geq 0, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m.
\end{align*}
\]

(7)

In (7), the first constraint represents the balance of supply and demand, i.e., the total quantity that facilities supply to a customer equals to the quantity that the customer requires, and the second constraint means the quantity that a facility supplies to customers should be no more than its capacity. For convenience, we denote the feasible allocation set by

\[
Z(\omega, \theta) = \left\{ z \in \mathbb{R}_+^{nm} \mid \begin{align*}
\sum_{j=1}^{n} z_{ij} &= \xi_j(\omega, \theta), \quad j = 1, 2, \ldots, m \\
\sum_{i=1}^{m} z_{ij} &\leq s_i, \quad i = 1, 2, \ldots, n \\
\sum_{i=1}^{m} z_{ij} &\geq 0, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m
\end{align*} \right\}.
\]

(8)

Note that \(Z(\omega, \theta)\) may be an empty set for some \((\omega, \theta)\).

For each \((\omega, \theta) \in \Omega \times \Theta\), the minimal transportation cost is the one associated with the best allocation \(z\), i.e.,

\[
\mathit{C}(x, y; \omega, \theta) = \min_{z \in Z(\omega, \theta)} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}.
\]

(9)

whose optimal solution \(z^*\) is called the optimal allocation. If \(Z(\omega, \theta) = \emptyset\), then the demands of some customers are impossible to be met. As a penalty, we define

\[
C(x, y; \omega, \theta) = \sum_{i=1}^{n} \max_{j \in \{1, \ldots, m\}} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}.
\]

(10)

Note that the minimal transportation cost \(C(x, y; \omega, \theta)\) defined by (8)-(10) is also a fuzzy random variable defined on \((\Omega, \mathcal{A}, \mathbb{P})\times(\Theta, \mathbb{P}(\Theta), \mathbb{P})\).

In the following three sections, three fuzzy random programming models will be presented for the capacitated location-allocation problem to minimize the transportation cost \(C(x, y; \omega, \theta)\).

4. Expected cost minimization model

The first type of the fuzzy random programming model is the so-called fuzzy random expected value model initiated by Liu and Liu (2003a), which selects the decision with optimal values of expected objective functions subject to some expected constraints. Since the minimal transportation cost \(C(x, y; \omega, \theta)\) is a fuzzy random variable, in order to evaluate the location design, we use the expected cost

\[
E[C(x, y; \omega, \theta)] = \int_{0}^{+\infty} \mathit{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta \mid C(x, y; \omega, \theta) \geq r \} \mathrm{d}r
\]

(11)

according to the definition of the expected value operator of fuzzy random variables in (4). Based on this idea, a fuzzy random expected cost minimization model for the capacitated location-allocation problem can be provided as follows,

\[
\begin{align*}
\min_{x, y} & \int_{0}^{+\infty} \mathit{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta \mid C(x, y; \omega, \theta) \geq r \} \mathrm{d}r \\
\text{subject to:} & \quad g_k(x, y) \leq 0, \quad k = 1, 2, \ldots, p
\end{align*}
\]

(12)

where \(\mathit{Ch}^M\) is the mean chance operator defined in (1), \(C(x, y; \omega, \theta)\) is defined by (8)-(10), and \(g_k(x, y) \leq 0, k = 1, 2, \ldots, p\) represent the potential regions for locating facilities. Model (12) means to minimize the expected cost \(E[C(x, y; \omega, \theta)]\) subject to some location region constraints.

5. \(z\)-Cost minimization model

As the second fuzzy random programming constructed by Liu (2001a), the chance-constrained programming satisfies the requirements for risk measurement, and offers a powerful approach of modeling fuzzy random decision systems with assumptions that the chance constraints hold at least with some predetermined confidence levels, which are provided as appropriate safety margins by the decision-maker. On the basis of such philosophy, instead of optimizing the transportation cost \(C(x, y; \omega, \theta)\) directly, in this problem we first define the \(z\)-cost as the minimum value \(\overline{T}\) such that

\[
\mathit{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta \mid C(x, y; \omega, \theta) \leq \overline{T} \} \geq z.
\]

Then we choose to constrain the chance of the event \(C(x, y; \omega, \theta) \leq \overline{T}\) larger than an acceptable confidence level \(z\) and simultaneously minimize the \(z\)-cost. For this purpose, we have a fuzzy random \(z\)-cost minimization model,

\[
\begin{align*}
\min_{x, y} & \overline{T} \\
\text{subject to:} & \quad \mathit{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta \mid C(x, y; \omega, \theta) \leq \overline{T} \} \geq z \\
& \quad g_k(x, y) \leq 0, \quad k = 1, 2, \ldots, p
\end{align*}
\]

(13)

where \(z\) is the predetermined confidence level, \(\overline{T}\) is the \(z\)-cost, and \(C(x, y; \omega, \theta)\) is defined by (8)-(10).
6. Chance maximization model

Sometimes, in practice, the total transportation cost is required to be less than a given cost supremum \( C^0 \) for the sake of budget limitation. For this purpose, the decision-maker desires to maximize the chance of satisfying the event \( C(x, y)(\omega, \theta) \leq C^0 \). In order to model this type of decision system, Liu (2001b) provided a theoretical framework of the fuzzy random dependent-chance programming, in which the underlying philosophy is based on selecting the decision with maximal chance to meet the event. As for the capacitated location-allocation problem, if we want to maximize the mean chance that the transportation cost does not exceed a predetermined cost supremum \( C^0 \), then we have a chance maximization model as follows,

\[
\begin{align*}
\max & \quad \text{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta : C(x, y)(\omega, \theta) \leq C^0 \} \\
\text{subject to:} & \quad g_k(x, y) \leq 0, \ k = 1, 2, \ldots, p
\end{align*}
\]

(14)

where \( C(x, y)(\omega, \theta) \) is defined by (8)–(10), and \( C^0 \) is a given supremum of the total transportation cost.

Until now, three fuzzy random programming models (12)–(14) have been built for the capacitated location-allocation problem. These models are different from the traditional fuzzy random programming models because for any sample \((\omega, \theta) \in \Omega \times \Theta\) satisfying \( Z(\omega, \theta) \neq \emptyset \), there is a sub-optimal problem in them, i.e.,

\[
\begin{align*}
\min & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} z_{ij} \left( x_i - a_j \right)^2 + \left( y_i - b_j \right)^2 \\
\text{subject to:} & \quad \sum_{i=1}^{m} z_{ij} = \xi_j(\omega, \theta), \quad j = 1, 2, \ldots, m \\
& \quad \sum_{j=1}^{n} z_{ij} \leq s_i, \quad i = 1, 2, \ldots, n \\
& \quad z_{ij} \geq 0, \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m.
\end{align*}
\]

(15)

Note that in (15) the parameters \( x_i, y_j, \) and \( \xi_j(\omega, \theta), i = 1, 2, \ldots, n, j = 1, 2, \ldots, m, \) are all fixed real numbers. It is clearly a linear programming problem, whose optimal objective value is \( C(x, y)(\omega, \theta) \).

7. Hybrid intelligent algorithms

Generally speaking, it is difficult to obtain results from fuzzy random programming models by traditional methods. A good way is to design some hybrid intelligent algorithms for figuring out some sample \((\omega, \theta) \in \Omega \times \Theta\) satisfying \( Z(\omega, \theta) \neq \emptyset \), there is a sub-optimal problem in them, i.e.,

\[
\begin{align*}
\min & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} z_{ij} \left( x_i - a_j \right)^2 + \left( y_i - b_j \right)^2 \\
\text{subject to:} & \quad \sum_{i=1}^{m} z_{ij} = \xi_j(\omega, \theta), \quad j = 1, 2, \ldots, m \\
& \quad \sum_{j=1}^{n} z_{ij} \leq s_i, \quad i = 1, 2, \ldots, n \\
& \quad z_{ij} \geq 0, \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m.
\end{align*}
\]

7.1. Computing optimal allocations

Assume that the locations \((x_i, y_j), i = 1, 2, \ldots, n, \) of the facilities have been given. For each \((\omega, \theta) \in \Omega \times \Theta\), we have the realized demands \( \xi_j(\omega, \theta), j = 1, 2, \ldots, m. \) In order to meet them with minimal cost, we should determine the optimal allocations. In other words, when \( Z(\omega, \theta) \neq \emptyset \), we need to solve the linear programming (15), which is actually a linear transportation problem for each realization of demands \( \xi_j(\omega, \theta), j = 1, 2, \ldots, m. \) Thus the network simplex algorithm is employed to obtain the optimal solution of model (15). We describe the algorithm simply by the following procedures. For detailed expositions, the interested reader may consult Bazaraa, Jarvis, and Sherali (1990).

Algorithm 1. (Network Simplex Algorithm)

\textbf{Step 1.} Find a starting basic feasible solution \( z_{ij}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, m, \) by the method of minimum elements.

\textbf{Step 2.} Compute the inspection values \( \sigma_u \) of all nonbasic variables, \( i = 1, 2, \ldots, n, j = 1, 2, \ldots, m, \) by the dual variable method.

\textbf{Step 3.} Suppose that \( \sigma_u \) is the least one in \( \{ \sigma_u, i = 1, 2, \ldots, n, j = 1, 2, \ldots, m \}. \) If \( \sigma_u \leq 0 \), then select the column \((k, l)\) to enter the basis, adjust the values of the variables around the cycle according to the sign of the coefficient in the representation, and then go to Step 2. Otherwise, return \( \sum_{i=1}^{m} \sum_{j=1}^{n} z_{ij} \leq 0, \) \( \sum_{j=1}^{n} z_{ij} \leq 0, \) and \( z_{ij} \geq 0, \) \( i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m. \)

7.2. Computing uncertain functions

By uncertain functions we mean the functions with fuzzy random parameters, just as those in models (12)–(14). Due to the complexity, we design some fuzzy random simulations to calculate the uncertain functions in this paper. The first type of the uncertain function is

\[
U_1 : (x, y) \rightarrow \int_0^{+\infty} \text{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta : C(x, y)(\omega, \theta) \geq r \} \, dr,
\]

(16)

which is the expected transportation cost in model (12), and can be represented as the following equivalent form based on (5),

\[
U_1 : (x, y) \rightarrow \int_0^{+\infty} \text{Pr} \{ (\omega) \in \Omega : \mathbb{E} [C(x, y)(\omega, \theta)] \geq r \} \, dr.
\]

(17)

In order to compute the expected cost \( U_1 \), we design a fuzzy random simulation by integrating the fuzzy simulation and the stochastic simulation as follows.

Algorithm 2. (Fuzzy Random Simulation for Expected Cost)

\textbf{Step 1.} Set \( U_1 = 0. \)

\textbf{Step 2.} Generate \( \omega \) from \( \Omega \) according to the probability measure \( \text{Pr}. \)

\textbf{Step 3.} Let \( U_1 \leftarrow U_1 + \mathbb{E} [C(x, y)(\omega, \theta)] \), where \( \mathbb{E} [C(x, y)(\omega, \theta)] \) is calculated by fuzzy simulation, and the optimal transportation costs \( C(x, y)(\omega, \theta) \) are obtained by solving the linear programming (15) via the network simplex algorithm (Algorithm 1).

\textbf{Step 4.} Repeat the second to third steps for \( N \) times, where \( N \) is a sufficiently large number.

\textbf{Step 5.} Return \( U_1/N. \)

The second type of the uncertain function is

\[
U_2 : (x, y) \rightarrow \text{Ch}^M \{ (\omega, \theta) \in \Omega \times \Theta : C(x, y)(\omega, \theta) \leq C^0 \},
\]

(18)

which is the mean chance of the fuzzy random event in model (14). It follows from the definition of the mean chance in (1) that \( U_2 \) has an equivalent form as follows,

\[
U_2 : (x, y) \rightarrow \int_0^1 \text{Pr} \{ \omega \in \Omega : \mathbb{C} \{ C(x, y)(\omega, \theta) \leq C^0 \} \geq \beta \} \, d\beta.
\]

(19)
Combining the fuzzy simulation and the stochastic simulation, we design the following fuzzy random simulation to obtain the mean chance $U_2$.

**Algorithm 3. (Fuzzy Random Simulation for Mean Chance)**

1. **Step 1.** Set $N = 0$.
2. **Step 2.** Generate $\beta \in [0, 1]$ uniformly.
3. **Step 3.** Generate $\omega$ from $\Omega$ according to the probability measure $P_{\omega}$.
4. **Step 4.** Compute the credibility $C_r \{ C(x, y|\omega, \theta) \leq C^0 \}$ by fuzzy simulation, where the optimal transportation cost $C(x, y|\omega, \theta)$ is obtained by solving the linear programming (15) via the network simplex algorithm (Algorithm 1).
5. **Step 5.** If $C_r \{ C(x, y|\omega, \theta) \leq C^0 \} \geq \beta$, then $N = N + 1$.
6. **Step 6.** Repeat the third to fifth steps for $N_1$ times, where $N_1$ is a sufficiently large number.
7. **Step 7.** Repeat the second to sixth steps for $N_2$ times, where $N_2$ is a sufficiently large number.
8. **Step 8.** Return $N/(N_1 N_2)$.

The third type of the uncertain function is

$$U_3 : (x, y) \rightarrow \min \left\{ f_j | \text{Ch}^{11} \{ (\omega, \theta) \in \Omega \times \Theta | C(x, y|\omega, \theta) \leq f_j \} \geq x \right\},$$

which is the $x$-cost in the chance-constrained programming model (13). According to the definition of the mean chance in (11), it is required to find the minimal value of $f$ satisfying

$$\int_0^1 \Pr \{ \omega \in \Omega | C_r \{ C(x, y|\omega, \theta) \leq f \} \geq \beta \} d\beta \geq x.$$

The following procedure is provided to seek for the $x$-cost $U_3$ based on the golden section search of optimization method in Kiefer (1953) and Algorithm 3.

**Algorithm 4. (Fuzzy Random Simulation for x-Cost)**

1. **Step 1.** Initialize the potential interval $[a_1, b_1] = [0, N]$, and set a small number $\epsilon > 0$ and the iteration counter $k = 0$, where $N$ is a sufficiently large number.
2. **Step 2.** Calculate $f_1^3 = a_1 + 0.382(b_1 - a_1)$ and $f_2^3 = a_1 + 0.618(b_1 - a_1)$.
3. **Step 3.** Compute the mean chance $\text{Ch}^{10}_i$ of the fuzzy random event $C(x, y|\omega, \theta) \leq f_i^3, i = 1, 2$, respectively, by utilizing Algorithm 3.
4. **Step 4.** Update the previous interval to $[a_{k+1}, b_{k+1}]$ by

$$[a_{k+1}, b_{k+1}] = \begin{cases} [a_1, f_1^3], & \text{if } \text{Ch}^{10}_1 > x \\ [f_2^3, f_2^3], & \text{if } \text{Ch}^{10}_2 < x < \text{Ch}^{10}_1 \\ [f_2^3, b_1], & \text{if } \text{Ch}^{10}_2 > x. \end{cases}$$

5. **Step 5.** Increment $k$ until $b_{k+1} - a_{k+1} < \epsilon$.
6. **Step 6.** Return $f_2^3$ as the $x$-cost $U_3$.

### 7.3. Hybrid intelligent algorithms

Several hybrid intelligent algorithms have been designed to work out the capacitated location-allocation problems with stochastic or fuzzy demands in Zhou and Liu (2003), Zhou and Liu (2007). Motivated by the philosophies in the literature above, in this section, some hybrid intelligent algorithms integrating the network simplex algorithm, fuzzy random simulation, and genetic algorithm are presented for solving models (12)–(14). The core idea of the algorithms is to embed the network simplex algorithm (Algorithm 1) and the fuzzy random simulations (Algorithms 2–4) into the genetic algorithm. The procedures of the hybrid intelligent algorithms designed in this paper are set out as follows.

**Algorithm 5. (Hybrid Intelligent Algorithms)**

1. **Step 1.** Initialize chromosomes $V_i = (x', y')$ for $i = 1, 2, \ldots, \text{pop}_i$, from the potential location regions $\{g_k(x, y) \leq 0, k = 1, 2, \ldots, p\}$ uniformly, where $\text{pop}_i$ is the number of chromosomes in the population.
2. **Step 2.** Calculate the objective values for all chromosomes $V_i, i = 1, 2, \ldots, \text{pop}_i$, by the designed fuzzy random simulations (Algorithms 2–4) for computing the uncertain functions, in which the network simplex algorithm (Algorithm 1) is used to solve the linear programming (15) for obtaining the optimal allocations.
3. **Step 3.** Compute the fitness of all chromosomes $V_i, i = 1, 2, \ldots, \text{pop}_i$, by the rank-based evaluation function $\text{Eval}(V_i) = a(i - a)^{-1}, i = 1, 2, \ldots, \text{pop}_i$, where $a \in (0, 1)$ is a parameter in the genetic system.
4. **Step 4.** Select $\text{pop}_i$ chromosomes for a new population by spinning the roulette wheel.
5. **Step 5.** Update chromosomes $V_i, i = 1, 2, \ldots, \text{pop}_i$, by crossover and mutation operations, and the feasibility of offsprings should be checked by the region constraints $g_k(x, y) \leq 0, k = 1, 2, \ldots, p$.
6. **Step 6.** Repeat the second to fifth steps for a given number of iterations.
7. **Step 7.** Report the best chromosome $V' = (x', y')$ found according to the fitness values as the optimal location.

### 8. Numerical examples

In this section, some numerical examples of a specific container freight station problem are used to show the feasibility and effectiveness of the proposed hybrid intelligent algorithms implemented in Microsoft Visual C++ 6.0 on a personal computer with Intel Core 2 Duo 2.10 GHz CPU and 1 GB memory.

Consider an international logistics enterprise who intends to reasonably locate some container freight stations with capacity limitations as logistics centers to connect container terminals and customers, and distribute cargoes from container terminals to customers. The container freight stations will be chosen from eastern coastal regions within about 100 km of the coast as shown in Fig. 2, with the purpose of minimizing the transportation cost from container freight stations to customers.

Suppose the enterprise serves 20 customers nationwide whose locations $(a_j, b_j), j = 1, 2, \ldots, 20$, are given in Table 1. The demands of the customers, i.e., $z_j, j = 1, 2, \ldots, 20$, are triangular-type fuzzy random variables given in Table 1, where $\rho_1 \sim N(5, 1)$ is a normally distributed variable, and $\rho_2 \sim U(8, 12)$ is a uniformly distributed variable. Besides, we assume that there are 4 container freight stations to be determined by the enterprise, whose capacities $s_i, i = 1, 2, 3, 4$, are given as 120, 150, 170 and 170, respectively, in the potential eastern coastal regions $10 \leq x_i \leq 110, 10 \leq y_i \leq 110, i = 1, 2, 3, 4$. 

...
Example 1. If we want to minimize the expected transportation cost, then we have the following expected value model for the container freight station problem:

\[
\min \int_0^\infty \mathbb{E}\left\{(\omega, \theta) \in \Omega \times \Theta \mid C(x, y(\omega, \theta)) \geq r\right\} dr
\]

subject to:

10 \leq x_i \leq 110, \quad i = 1, 2, 3, 4
10 \leq y_i \leq 110, \quad i = 1, 2, 3, 4

where

\[
C(x, y(\omega, \theta)) = \begin{cases} \min_{x, y} \sum_{j=1}^{20} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}, & \text{if} \ Z(\omega, \theta) \neq \emptyset \\
\sum_{j=1}^{20} \max_{i \in \Theta} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}, & \text{otherwise} \end{cases}
\]

and

\[
Z(\omega, \theta) = \begin{cases} \sum_{i=1}^{20} z_i, & j = 1, 2, \ldots, 20 \\
z_i \geq 0, & i = 1, 2, 3, 4, \quad j = 1, 2, \ldots, 20 \end{cases}
\]

By performing 1000 \times 1000 cycles in fuzzy random simulation, the hybrid intelligent algorithm (Algorithm 5) proposed in Section 7 is run with 1000 generations. Different groups of parameters in genetic algorithm system are adopted for comparison, and the results for each group are shown in Table 2, where \(P_c\) and \(P_m\) are the probabilities of crossover and mutation, respectively, \(a\) is the parameter in the evaluation function, and “Cost” is the minimal expected cost.

From Table 2 it appears that all the minimal expected costs differ a little from each other. In order to account for the differentia among them, we present a parameter, called the percent error, i.e., (actual value - optimal value)/optimal value \times 100\%, where the optimal value is the least one of all the ten minimal expected costs obtained. The percent error is named by “Error” in the last column of Table 2. It follows from Table 2 that the percent error does not exceed 2.28\% when different parameters are selected, which implies that the hybrid intelligent algorithm is robust to the parameter settings, and effective in solving the expected value model (21) for the container freight station problem. Finally, we choose the least one of all the ten minimal expected costs in Table 2 as the optimal objective value of model (21), i.e., 7103, whose corresponding optimal location is

![Fig. 2. A container freight station problem.](image-url)
### Table 2
Solutions comparison of Example 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>pop size</th>
<th>$p_c$</th>
<th>$p_m$</th>
<th>$a$</th>
<th>Optimal locations</th>
<th>Cost</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.1</td>
<td>0.2</td>
<td>0.05</td>
<td>(16.79, 18.74), (58.80, 73.05), (76.73, 19.49), (30.91, 54.17)</td>
<td>7265</td>
<td>2.28</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.3</td>
<td>0.2</td>
<td>0.05</td>
<td>(18.29, 20.05), (59.68, 72.27), (77.62, 20.54), (30.50, 53.94)</td>
<td>7228</td>
<td>1.76</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.1</td>
<td>0.3</td>
<td>0.10</td>
<td>(19.13, 18.35), (58.49, 73.41), (75.24, 21.06), (31.24, 54.73)</td>
<td>7202</td>
<td>1.39</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>0.3</td>
<td>0.1</td>
<td>0.08</td>
<td>(18.52, 20.25), (57.99, 72.79), (75.35, 21.32), (30.89, 52.98)</td>
<td>7186</td>
<td>1.17</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>0.2</td>
<td>0.2</td>
<td>0.10</td>
<td>(17.29, 18.80), (58.90, 73.12), (76.46, 20.75), (31.90, 55.23)</td>
<td>7179</td>
<td>1.07</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>0.2</td>
<td>0.2</td>
<td>0.10</td>
<td>(18.38, 19.21), (59.94, 72.53), (77.56, 19.39), (31.31, 53.66)</td>
<td>7170</td>
<td>0.94</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>0.3</td>
<td>0.1</td>
<td>0.08</td>
<td>(17.57, 20.18), (58.52, 73.08), (76.89, 20.96), (31.12, 54.43)</td>
<td>7135</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>0.1</td>
<td>0.2</td>
<td>0.05</td>
<td>(18.04, 19.52), (59.96, 72.24), (77.35, 19.39), (31.45, 53.66)</td>
<td>7127</td>
<td>0.34</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>0.3</td>
<td>0.2</td>
<td>0.05</td>
<td>(18.28, 19.51), (59.94, 71.85), (75.82, 20.81), (31.18, 54.59)</td>
<td>7119</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>0.1</td>
<td>0.3</td>
<td>0.10</td>
<td>(18.05, 19.95), (60.19, 71.84), (77.82, 19.82), (31.67, 53.93)</td>
<td>7103</td>
<td>0.00</td>
</tr>
</tbody>
</table>

![Fig. 3. Illustration of expected costs with regard to iterations.](image)

![Fig. 4. Illustration of 0.9-costs with regard to iterations.](image)

### Table 3
Solutions comparison of Example 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>pop size</th>
<th>$p_c$</th>
<th>$p_m$</th>
<th>$a$</th>
<th>Optimal locations</th>
<th>Cost</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.1</td>
<td>0.2</td>
<td>0.05</td>
<td>(18.50, 19.31), (28.54, 53.32), (73.32, 19.87), (58.29, 61.56)</td>
<td>7519</td>
<td>2.51</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.1</td>
<td>0.3</td>
<td>0.10</td>
<td>(18.42, 20.51), (27.15, 54.33), (76.13, 19.78), (60.11, 59.08)</td>
<td>7502</td>
<td>2.28</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>0.3</td>
<td>0.1</td>
<td>0.08</td>
<td>(17.91, 20.43), (28.65, 54.55), (78.54, 20.45), (58.21, 60.87)</td>
<td>7496</td>
<td>2.19</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.3</td>
<td>0.2</td>
<td>0.05</td>
<td>(18.94, 19.22), (28.75, 55.26), (78.12, 20.76), (59.37, 61.53)</td>
<td>7470</td>
<td>1.84</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>0.2</td>
<td>0.2</td>
<td>0.10</td>
<td>(18.36, 19.89), (28.87, 53.90), (77.09, 18.27), (59.22, 60.36)</td>
<td>7453</td>
<td>1.61</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>0.2</td>
<td>0.2</td>
<td>0.10</td>
<td>(19.46, 20.22), (29.42, 54.59), (76.11, 20.85), (60.67, 59.39)</td>
<td>7441</td>
<td>1.45</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>0.3</td>
<td>0.2</td>
<td>0.05</td>
<td>(19.24, 20.12), (28.27, 53.18), (77.58, 19.80), (59.63, 60.52)</td>
<td>7428</td>
<td>1.27</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>0.3</td>
<td>0.1</td>
<td>0.08</td>
<td>(18.62, 19.88), (28.58, 53.21), (77.31, 20.47), (61.20, 59.95)</td>
<td>7417</td>
<td>1.12</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>0.1</td>
<td>0.2</td>
<td>0.05</td>
<td>(17.35, 21.01), (27.78, 55.25), (76.29, 19.99), (60.91, 59.75)</td>
<td>7384</td>
<td>0.67</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>0.1</td>
<td>0.3</td>
<td>0.10</td>
<td>(18.91, 20.05), (28.26, 54.90), (77.15, 21.02), (58.15, 61.29)</td>
<td>7335</td>
<td>0.00</td>
</tr>
</tbody>
</table>
(x’, y’) = ([(18.05, 19.95), (60.19, 71.84), (77.82, 19.82), (31.67, 53.93)].

In addition, we illustrate the convergence rate of the solution (x’, y’) in Fig. 3 where the objective values of the solutions obtained after around 100 iterations have already been approximately equal to that of the optimal solution.

Example 2. If we want to minimize the 0.9-cost, we have the following chance-constrained programming model for the container freight station problem,

\[
\begin{align*}
\min_{x, y} & \quad \text{Ch}^M \left\{ (\omega, \theta) \in \Omega \times \Theta \mid C^M(x, y; \omega, \theta) \leq 7500 \right\} \\
\text{subject to:} & \\
10 \leq x_i \leq 110, & i = 1, 2, 3, 4 \\
10 \leq y_i \leq 110, & i = 1, 2, 3, 4
\end{align*}
\]

where \( C^M(x, y; \omega, \theta) \) is given by (22) and (23).

A numerical study is also carried out to compare the solutions obtained by running the hybrid intelligent algorithm with various parameters, and all the computational results are shown in Table 3, where “Cost” is the minimal 0.9-cost, and the cycle number in the fuzzy random simulation is 1000 × 1000.

Solutions with different parameters of genetic algorithm are compared on the basis of equivalent generations. In order to measure the difference of each other, the percent error is proposed and shown in Table 3 as “Error”. It follows from Table 3 that the maximal percent error does not exceed 2.51% when different parameters are used. Thus, the hybrid intelligent algorithm is also robust to the parameter settings, and effective to solve the chance-constrained programming model (24) for the container freight station problem. In conclusion, we choose the least one of all the ten minimal 0.9-costs in Table 3 as the optimal objective value of model (24), i.e., 7335, whose corresponding optimal location is

\[(x’, y’) = ([(18.91, 20.05), (28.26, 54.90), (77.15, 21.02), (58.15, 61.29)])

A convergence process of this solution is demonstrated in Fig. 4 as a representative, from which it is obvious that by the hybrid intelligent algorithm, solutions can converge to the optimal one quickly.

Example 3. In order to maximize the mean chance that the total transportation costs do not exceed 7500, we have a chance maximization model for the container freight station problem as follows,

\[
\begin{align*}
\max_{x, y} & \quad \text{Ch}^M \left\{ (\omega, \theta) \in \Omega \times \Theta \mid C^M(x, y; \omega, \theta) \leq 7500 \right\} \\
\text{subject to:} & \\
10 \leq x_i \leq 110, & i = 1, 2, 3, 4 \\
10 \leq y_i \leq 110, & i = 1, 2, 3, 4
\end{align*}
\]

where \( C^M(x, y; \omega, \theta) \) is given by (22) and (23).

The hybrid intelligent algorithm is run with 1000 × 1000 cycles in fuzzy random simulation and 1000 generations in genetic algorithm. Similarly, we run the hybrid intelligent algorithm for ten times with different parameters of genetic algorithm on the basis of equivalent generations, and corresponding solutions are given in Table 4, where “MChan.” is the maximal mean chance.

In order to measure the difference between these results, “Error”, i.e., the percent error, is calculated and given in Table 4. From these computational results, we see that the maximal percent error does not exceed 2.59% when different parameters

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig_5.png}
\caption{Illustration of mean chances with regard to iterations.}
\end{figure}
are chosen. Therefore, the hybrid intelligent algorithm is also robust to the parameter settings and effective to solve the dependent-chance programming model (25) for the container freight station problem.

Finally, we choose the largest one of all the ten maximal mean chances in Table 4 as the optimal objective value of model (25), i.e., 96.4%, whose corresponding optimal location is $(x^*, y^*) = ((18.25, 19.23), (58.94, 72.53), (77.23, 19.26), (30.75, 53.96))^T$.

The convergence procedure of this solution is illustrated in Fig. 5, where the solutions converge to the optimal solution in around 400 iterations.

9. Conclusions

In this paper, we have contributed to the research area of the location-allocation problem in the following three aspects: (i) we discussed the capacitated location-allocation problem in the fuzzy random environment based on the mean chance; (ii) utilizing the mean chance, three types of fuzzy random programming models, i.e., the expected cost minimization model, the $\alpha$-cost minimization model, and the chance maximization model, were proposed to formulate the capacitated location-allocation problem; (iii) in order to solve these models efficiently, we integrated the network simplex algorithm, fuzzy random simulation, and genetic algorithm to produce some hybrid intelligent algorithms, which were illustrated by numerical examples about a container freight station problem.

The primitive chance presented in 2001 was often used to deal with the fuzzy random related problems in the previous literature, while the mean chance later proposed in 2005 was seldom employed. As an advanced version of chance measure, the mean chance has more strength than the primitive chance because of its scalar nature. Thereby, it has sufficient advantages to own wider applications in the future work.

Acknowledgments

This work was supported by grants from the National Social Science Foundation of China (No. 13CCGL057), the National Natural Science Foundation of China (No. 71272177), and the Ministry of Education Funded Project for Humanities and Social Sciences Research (No. 12JDXF005).

References